PPOL502-01, Spring 2016
Course Notes # 12: Functional Form - Quadratics

I. FUNCTIONAL FORM OVERVIEW

- "Functional form" refers to, well, the functional form of the variables in the model: are they in level form? in natural log? squared? cubed? square root? indicators? interactions? something else

- And the double whammy: when a particular form of a variable should be in a model but is not, the problem is referred to as "functional form misspecification." This is like an omitted variable problem.

- Over the next few lectures, we'll focus on WHY and HOW to transform the functional form of variables in a model, and we'll also talk some more about model specification – e.g., how do we know if we have functional form misspecification? We'll start with some special functional form transformations….

- Why do we care about functional form? remember, we're estimating a *linear* model (in the parameters) and sometimes we need to account for nonlinear relationships between the Xs and Y (think of the pictures of the logarithmic transformations earlier in the semester).

II. LOGARITHMIC FUNCTIONAL FORMS

- We talked about logarithmic forms a few months ago.

- Please review your notes from earlier regarding the interpretation of these models.

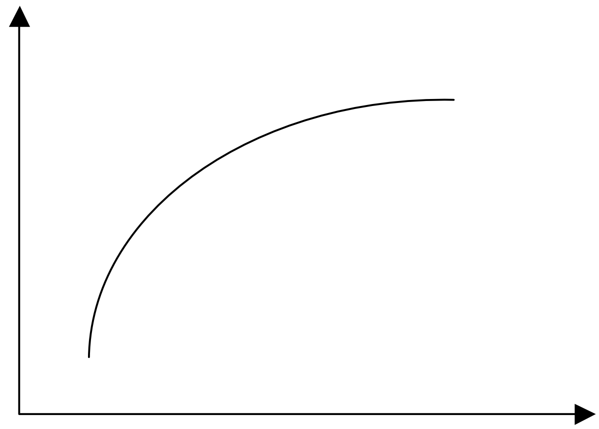
III. QUADRATIC TERMS

- A *quadratic* indicates a squared form of a variable (X^2). A *cubic* indicates a cubed term (X^3), etc.

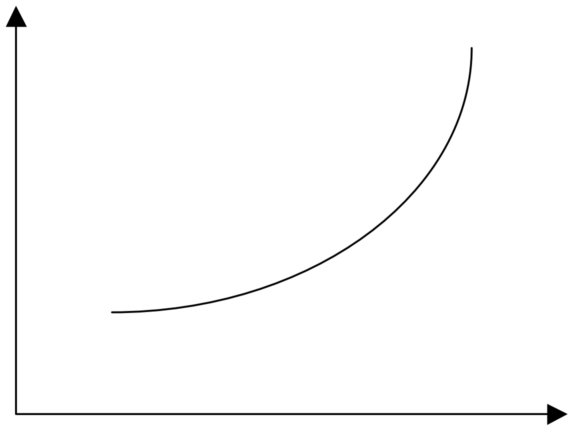
- In models that include quadratics, cubes, etc., it is standard to include all prior transformations also (e.g., if you include X^2, you should also include X; if you include X^3, you should also include X and X^2.

General Model: $\hat{Y} = \hat{\beta}_0 + \hat{\beta}_1 X_1 + \hat{\beta}_2 X_1^2$

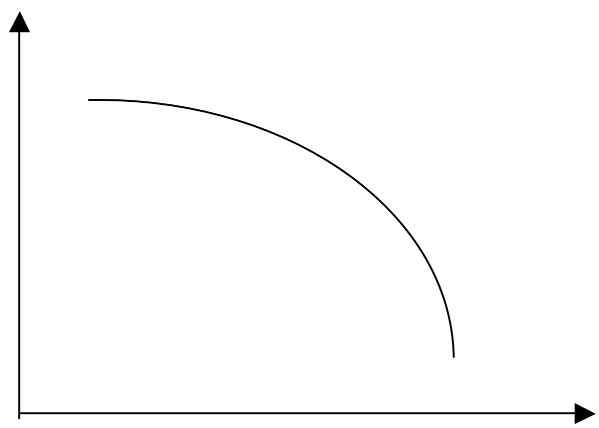
Pictures:



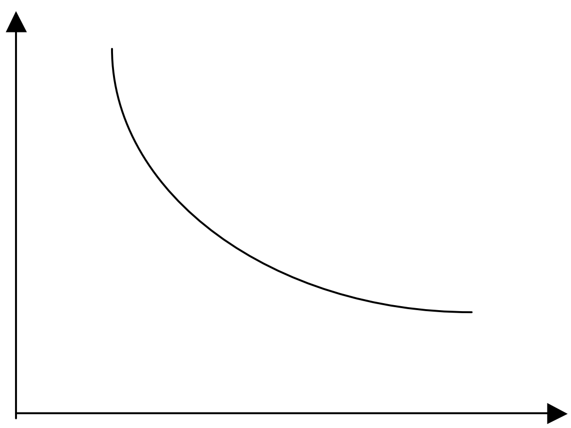
$\hat{\beta}_1$ *is* + $\quad$ $\hat{\beta}_2$ *is* −



$\hat{\beta}_1$ *is* + $\quad$ $\hat{\beta}_2$ *is* +



$\hat{\beta}_1$ *is* − $\quad$ $\hat{\beta}_2$ *is* −



$\hat{\beta}_1$ *is* − $\quad$ $\hat{\beta}_2$ *is* +

(data set is CARD.RAW)

```
. sum wage IQ exper expersq

    Variable |       Obs        Mean    Std. Dev.       Min        Max
-------------+--------------------------------------------------------
        wage |      3010    577.2824    262.9583        100       2404
          IQ |      2061    102.4498    15.42376         50        149
       exper |      3010    8.856146    4.141672          0         23
     expersq |      3010    95.57907    84.61831          0        529
```

```
. reg wage IQ exper
```

```
      Source |       SS       df       MS              Number of obs =    2061
-------------+------------------------------           F(  2,  2058) =   86.71
       Model |  11148498.9     2  5574249.45           Prob > F      =  0.0000
    Residual |   132299572  2058  64285.5065           R-squared     =  0.0777
-------------+------------------------------           Adj R-squared =  0.0768
       Total |   143448071  2060  69634.9861           Root MSE      =  253.55

------------------------------------------------------------------------------
        wage |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
          IQ |   4.604898   .3876702    11.88   0.000     3.844631    5.365164
       exper |   15.29233   1.601728     9.55   0.000     12.15115     18.4335
       _cons |   13.48337   46.64525     0.29   0.773    -77.99344    104.9602
------------------------------------------------------------------------------
```

- <u>Interpretation of the coefficient on EXPER</u>**:** Holding IQ constant, each additional year of experience is associated with a 15.29 cent increase in hourly wages, on average.

- What wage do you predict for a person who has the following number of years of experience? (just use the mean value for IQ in the prediction):

2 years of experience?:	*WAGEHAT* = 13.48 + 4.60(102.45) + 15.29(2) = 515.33 cents/hr
3 years?	*WAGEHAT* = 13.48 + 4.60(102.45) + 15.29(3) = 530.62 cents/hr
20 years?	*WAGEHAT* = 13.48 + 4.60(102.45) + 15.29(20) = 790.55 cents/hr
21 years?	*WAGEHAT* = 13.48 + 4.60(102.45) + 15.29(21) = 805.84 cents/hr

- Now, we'll estimate a model that predicts wage as a function of IQ and experience, where the effects of experience are allowed to vary depending on the level of experience.

  -- One way to do this would be to include an indicator variable for each and every level of experience (except for one baseline category level). This approach requires no assumptions about functional form, but has the disadvantage of requiring many parameters to capture the effect of experience.

  -- An easier and more parsimonious way to model this is to include a squared term of experience, along with the EXPER variable:

```
. reg wage IQ exper expersq

      Source |       SS       df       MS              Number of obs =    2061
-------------+------------------------------           F(  3,  2057) =   64.42
       Model |  12318991.3     3  4106330.42           Prob > F      =  0.0000
    Residual |   131129080  2057  63747.7297           R-squared     =  0.0859
-------------+------------------------------           Adj R-squared =  0.0845
       Total |   143448071  2060  69634.9861           Root MSE      =  252.48

------------------------------------------------------------------------------
        wage |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
          IQ |   4.743112   .3873905    12.24   0.000     3.983394    5.502831
       exper |   41.57254   6.337069     6.56   0.000      29.1448    54.00028
     expersq |   -1.43637   .3352081    -4.29   0.000    -2.093753   -.7789877
       _cons |  -100.1911   53.49143    -1.87   0.061    -205.0941    4.711934
------------------------------------------------------------------------------
```

- In this regression specification, the effect of an extra year of experience depends on the level of experience. So, we can't say what the predicted effect of an extra year of experience is on wages, unless we specify exactly what level of experience we are talking about.

  To be precise, take the derivative of the estimated regression equation with respect to experience:

  $$\hat{Y} = -100.19 + 4.74IQ + 41.57EXPER - 1.44EXPER^2 \qquad and...$$

  $$\frac{\partial(\hat{Y})}{\partial EXPER} = 41.57 - (2)*(1.44)*(EXPER)$$

- And we can do full predictions of wage based on specific levels of experience:

- What wage do we predict now for a person who has

2 years of experience?: *WAGEHAT* = -100.19 + 4.74(102.45) + 41.57(2) –1.44(4) = 462.80 cents/hr
3 years? *WAGEHAT* = -100.19 + 4.74(102.45) + 41.57(3) –1.44(9) = 497.17 cents/hr
20 years? *WAGEHAT* =-100.19 + 4.74(102.45) + 41.57(20) –1.44(400) = 640.82 cents/hr
21 years? *WAGEHAT* = -100.19 + 4.74(102.45) + 41.57(21) –1.44(441) = 623.35 cents/hr

- So, what's the effect of an extra year of experience when that additional year is ***from 2 to 3 years***? We can use two methods. The first one uses the predictions we calculated above:

Method 1: (497.17 – 462.80) = 34.37 more cents/hr

(*Note*: if we would not have rounded the values of the EXPER and EXPERSQ coefficients, we would have calculated exactly a 34.39 cent/hr difference).

Method 2: How could we have estimated this *difference* in predicted wages using just the regression coefficient estimates (i.e., holding constant the values of the other variables in the model)?

$$\Delta WAGEHAT = 41.57254(\Delta EXPER) - (1.43637) * (\Delta EXPERSQ)$$

-- to implement this formula, we need to figure out what the correct values are for $\Delta EXPER$ and for $\Delta EXPERSQ$:

-- The change in experience (i.e. $\Delta EXPER$) in going from 2 to 3 years is just equal to 1 year of experience.

-- However, the change in experience-squared (i.e. $\Delta EXPERSQ$) in going from 2 to 3 years is 9 – 4 = 5 years of experience-squared:

:

$$\Delta WAGEHAT = 41.57254(1) - (1.43637) * (5) = 34.39 \ cents / hr$$

- O.K., let's look at another example: What's the effect of an extra year of experience when that additional year is ***from 20 to 21 years***?

  Method 1: (623.35 - 640.82) = -17.47 (Or 17.47 *fewer* cents/hr). If we would not have rounded the values of the EXPER and EXPER squared coefficients, we would have predicted wages of 642.33 and 625.01 respectively, for a difference of 17.32 fewer cents/hr.

  Method 2: Or, using the regression coefficients directly, we could have calculated the change in predicted wages as:

  -- The change in experience (i.e. $\Delta EXPER$) in going from 20 to 21 years is just equal to 1.

  -- However, the change in experience-squared (i.e. $\Delta EXPERSQ$) in going from 20 to 21 years is 441-400 = 41.

  -- So: $\Delta WAGEHAT = 41.57254(1) - (1.43637)*(41) = -17.32\ cents / hr$

Note: We can easily find the "turning point" at which an additional year of experience is predicted to lead to decreased wages by setting the partial derivative with respect to EXPER to zero:

$$\frac{\partial(\hat{Y})}{\partial EXPER} = 41.57 - (2)*(1.44)*(EXPER) = 0$$
$$= -2.88 EXPER = -41.57$$
$$= EXPER = \frac{41.57}{2.88} = 14.43$$

The general formula for computing the turning point is (see Wooldridge page 193):

$$x^* = |\hat{\beta}_1 / (2\ \hat{\beta}_2)|$$

Note that this formula is only relevant when one of the coefficients is positive and the other is negative because the curve does not change directions if both coefficients are the same sign (see pictures on first page of notes).

- So, the return to an additional year of experience becomes negative at 14.43 years of experience. At less than 14.43 years of experience, wages will increase when experience increases. At more than 14.43 years of experiences, wages will decrease when experience increases (try calculating it yourself! at home! and prove to yourself that what the quant swami says is true).

  (NOTE: Wooldridge p. 193-194 discusses why declining returns to experience in wage models might imply misspecification of the model.)

- If we simply ask the question: “conditional on IQ, does experience explain variation in wages,” then we have to do a test of joint significance on the coefficients for EXPER and EXPERSQ.

  -- What null is being tested?

  -- How do you interpret these results? Are you surprised? Why or why not?

```
. test exper = expersq = 0

 ( 1)  exper - expersq = 0
 ( 2)  exper = 0

       F(  2,  2057) =   55.14
            Prob > F =    0.0000
```

- **So what’s the point?**

  -- The estimated wages in using the squared term are lower for each level of experience, compared to the estimates using the coefficient estimates from the regression WITHOUT the squared term.

  -- The more important point to see here, however, is that the return (measured in wages/hr) to an extra year of experience depends on the level of experience. In the model with no squared term, an extra year of experience was predicted to increase wages by 15.29 cents/hour, no matter whether that additional year of experience came very early in a career (in this example, from 2 to 3 years), or later in a career (in this example, from 20 to 21 years).

  -- By including a squared term for experience along with the level form of experience, we allowed the effect of that extra year of experience to vary depending on the level of experience.

- Note: As Wooldridge pp. 194-195 notes, interpretation of squared explanatory variables in a model where Y is measured in logarithms requires some extra care in interpreting.

```
/****************************************************************************
PPOL 509
Course Notes 12: Quadratics
****************************************************************************/
cd "C:\...\PPOL509\Stata datasets"

capture: log close
log using "..\do files\notes15.txt", text replace
set more off

clear
use card.dta

sum wage IQ exper expersq

reg wage IQ exper
reg wage IQ exper expersq
test exper = expersq = 0

log close
```