PPOL 503-03, PPOL 503-04, Fall 2016
Course Notes # 3: Interactions and F Tests

I. INTERACTIONS: BASIC IDEA

- In multiple regression models, we interpret a particular regression coefficient as the "effect" of that *X* variable (e.g. of X_1) on *Y*, holding the other *X*s in the model constant. When we first introduced the idea of "holding constant," we said that this was restrictive in the sense that the effect of X_1 on *Y* **was constrained to be constant across the range of *other* X values.** (e.g., the effect of POLICE on CRIMES was constrained to be constant across the different values of CRIMINAL; yet in some of the specifications, we saw that the effect of police on crimes was in fact different across different levels of criminals; our model just didn't account for that difference).

- Using **interactions,** we have a way of allowing the effect of one variable, X_1 on *Y* to vary, depending on the value of another variable, X_2.

- We can have interactions between (1) two indicator variables; (2) an indicator and an interval/ratio variable; or (3) two interval ratio variables. The idea is the same: The interaction term allows the effect of one variable to vary, depending on the value of the other variable.

- It is possible to interact three or more variables; however, it can become difficult to interpret these coefficients. They are not used much in practice, and we won't cover them here.

II. INTERACTIONS WITH TWO INDICATOR VARIABLES

- First, let's start out with a simple model of *bwght* on *white* and *male*:

```
. reg bwght white male

      Source |       SS       df       MS              Number of obs =    1388
-------------+------------------------------           F(  2,  1385) =   15.40
       Model |   12499.855     2  6249.92752           Prob > F      =  0.0000
    Residual |  562111.865  1385  405.856942           R-squared     =  0.0218
-------------+------------------------------           Adj R-squared =  0.0203
       Total |   574611.72  1387  414.283864           Root MSE      =  20.146

------------------------------------------------------------------------------
       bwght |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       white |   6.365519   1.315637     4.84   0.000     3.784662    8.946376
        male |   3.057256   1.082694     2.82   0.005     .9333584    5.181153
       _cons |   112.1128   1.304413    85.95   0.000      109.554    114.6716
------------------------------------------------------------------------------
```

- This model constrains the "effect" of being MALE to be the same for WHITES as well as for NONWHITES. ("Holding race constant….")

- And, this model constraints the "effect" of being WHITE to be the same for MALES as well as FEMALES. ("Holding gender constant….")

- But, what if the effect of gender varies by the race of the baby (is the effect of gender different for whites and nonwhites)? Alternatively, does the effect of race vary by gender (is the effect of race different for males and females)? Another way we say this: is there a *gender*race* effect on *birthweight*?

- We can accomplish this in two ways:

First option: Include main effects for *white* and *male*, and one <u>interaction</u> term for *whmale:*

To compute an interaction term, simply multiply the two main effect variables:

```
. gen whmale = white*male

. reg bwght white male whmale

      Source |       SS       df       MS              Number of obs =    1388
-------------+------------------------------           F(  3,  1384) =   10.28
       Model |  12525.2107     3  4175.07023           Prob > F      =  0.0000
    Residual |  562086.509  1384  406.131871           R-squared     =  0.0218
-------------+------------------------------           Adj R-squared =  0.0197
       Total |   574611.72  1387  414.283864           Root MSE      =  20.153

------------------------------------------------------------------------------
       bwght |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       white |    6.71902   1.932267     3.48   0.001     2.928531    10.50951
        male |   3.575291   2.339114     1.53   0.127    -1.013301    8.163882
      whmale |  -.6594038    2.63905    -0.25   0.803    -5.836374    4.517567
       _cons |   111.8321   1.721763    64.95   0.000     108.4546    115.2097
------------------------------------------------------------------------------
```

<u>What is the baseline category?</u>

E.g.: predicted birthweight from this model for:

(a) female babies who are <u>not</u> white = **111.8 ounces**

(b) male babies who are <u>not</u> white = 111.8 + (3.6*1) = **115.4 ounces**

(c) female babies who are white = 111.8 + (6.7*1) = **118.5 oz**

(d) male babies who are white =111.8 + (6.7*1) + (3.6*1) – (0.66*1) = **121.4 oz**

Second option: We can use the two different indicator variables (e.g., male/nonmale and white/nonwhite) to create a mutually exclusive set of indicators:

```
. reg bwght whfemale nonwhmale whmale

      Source |       SS       df       MS              Number of obs =    1388
-------------+------------------------------           F(  3,  1384) =   10.28
       Model |  12525.2107     3  4175.07023           Prob > F      =  0.0000
    Residual |  562086.509  1384  406.131871           R-squared     =  0.0218
-------------+------------------------------           Adj R-squared =  0.0197
       Total |   574611.72  1387  414.283864           Root MSE      =  20.153

------------------------------------------------------------------------------
       bwght |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
    whfemale |    6.71902   1.932267     3.48   0.001     2.928531    10.50951
   nonwhmale |   3.575291   2.339114     1.53   0.127    -1.013301    8.163882
      whmale |   9.634906   1.920523     5.02   0.000     5.867456    13.40236
       _cons |   111.8321   1.721763    64.95   0.000     108.4546    115.2097
------------------------------------------------------------------------------
```

where: WHFEMALE = 1 if white female, =0 otherwise
NONWHMALE = 1 if nonwhite male, =0 otherwise
WHMALE = 1 if white male, =0 otherwise

- What is the baseline category for this conceptual set of indicators?
- Are the coefficients statistically significant?
- Interpret the coefficient on WHFEMALE:
- Interpret the coefficient on NONWHMALE:
- Interpret the coefficient on WHMALE:
- What is the predicted birthweight for

(a) nonwhite, female babies?

(b) white, female babies?

(c) nonwhite, male babies?

(d) white, male babies?

These predictions are exactly equal to those obtained under method 1 and both are equivalent to the group means:

```
. * Statistics by gender and race
. bys male white: summarize bwght

----------------------------------------------------------------------
-> male = 0, white = 0

    Variable |       Obs        Mean    Std. Dev.       Min        Max
-------------+--------------------------------------------------------
       bwght |       137    111.8321    20.12665         30        158


----------------------------------------------------------------------
-> male = 0, white = 1

    Variable |       Obs        Mean    Std. Dev.       Min        Max
-------------+--------------------------------------------------------
       bwght |       528    118.5511    20.16924         35        271


----------------------------------------------------------------------
-> male = 1, white = 0

    Variable |       Obs        Mean    Std. Dev.       Min        Max
-------------+--------------------------------------------------------
       bwght |       162    115.4074    22.18095         23        176


----------------------------------------------------------------------
-> male = 1, white = 1

    Variable |       Obs        Mean    Std. Dev.       Min        Max
-------------+--------------------------------------------------------
       bwght |       561     121.467    19.52113         50        192
```

- Note that the means above are exactly the same predicted birthweights that we got when we explicitly included the interaction term *whmale* in the model above under option 1.

III. INTERACTIONS WITH ONE INDICATOR AND ONE INTERVAL/RATIO VAR

- What if the "effect" of the indicator variable (e.g., race) on Y (birthweight) is not constant throughout the range of the ratio/interval variable (e.g., mother's age), or equivalently, the "effect" of the interval/ratio variable is not the same for both levels of the indicator variable?

  Using the data set **BWGHT2.RAW**, how can we allow for the effect of mother's age (*mage)* on baby's birthweight to vary by race (where the mother's race is indicated by *mwhte*). By including an interaction (*mwhmage)* between these two variables, we allow the effect of *mage* on *bwght* to be different for white and nonwhite mothers:

  -- To allow for this, we need to create an ***interaction term*** between the *mwhte* and *mage* variables and include this new interaction term in the equation:

```
. use bwght2.dta
.
. gen mwhmage = mwhte*mage
.
. reg bwght mage mwhte mwhmage

      Source |       SS       df       MS              Number of obs =    1832
-------------+------------------------------           F(  3,  1828) =    3.67
       Model |  3646382.99     3     1215461           Prob > F      =  0.0118
    Residual |   604984392  1828  330954.262           R-squared     =  0.0060
-------------+------------------------------           Adj R-squared =  0.0044
       Total |   608630775  1831  332403.481           Root MSE      =  575.29

------------------------------------------------------------------------------
       bwght |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
        mage |   18.57864    7.92029     2.35   0.019     3.044867     34.1124
       mwhte |   586.3524   251.3175     2.33   0.020     93.45283    1079.252
     mwhmage |  -16.82123   8.475335    -1.98   0.047    -33.44359    -.198877
       _cons |   2773.682   234.4771    11.83   0.000     2313.811    3233.553
------------------------------------------------------------------------------
```

- What is the baseline category?

- predicted birthweight for babies born to: **[note: do these predictions make sense to you?]**

  (a) nonwhite mothers who are zero years old = **2773.68 grams**

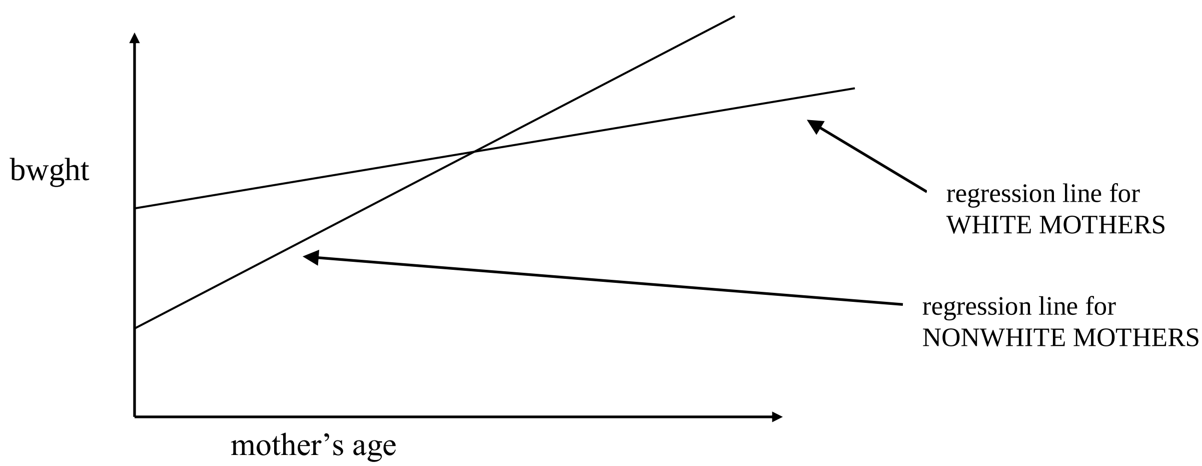
  (b) white mothers who are zero years old = 2773.68 + 586.35= **3360.03 grams**

  (c) nonwhite mothers who are 30 years old = 2773.68 + (18.58*30) = **3331.08 grams**

  (d) white mothers who are 30 years old = 2773.68 + (18.58*30) + 586.35 + (-16.82*30) = **3412.83 grams**

- The <u>interaction term allows the effect of mother's age to vary depending on race of the mother</u>. The coefficient estimate on the interaction $\hat{\beta}_{MWHMAGE}$ = -16.82, and it is statistically significant at conventional levels (*p*=0.0473). Because this interaction is significant, we can conclude that the effect of mother's age *does* vary by race. Alternatively, it could be interpreted as meaning that the effect of race on birthweight differs depending on the mother's age. This is like the "return to education" example in Wooldridge (p. 241) where he is interested in whether an extra year of education has the same effect for men as for women. In the birthweight example, we are interested in whether an additional year of age for the mother has the same effect on birthweight for whites compared to nonwhites (note that we are not controlling for anything else in this model: if we were, we would interpret the estimates as "holding constant the other variables in the model," as usual).

A picture….



- What is the <u>effect</u> of age on birthweight?

  Answer—it depends on the race (2 categories) of the mother:

  NONWHITE MOTHERS: 18.57864 (coeff for *mage*)
  WHITE MOTHERS: 18.57864 + (- 16.2123) = 2.36

Interpretation: We predict that birthweight will increase by about 18.6 grams for each additional year of mother's age for nonwhite mothers, but only by about 2.4 grams for white mothers.

- What is the <u>effect</u> of race on birthweight?

  Answer—it depends on the age of the mother (which ranges from 16 to 44 in the sample)

  Direct interpretation of *mwhte* coefficient:

    A baby born to a white mother who is zero years old (not possible, but hang tight!) has a predicted birthweight of 586 grams *more* than a baby born to a black mother who is zero years old. This is not very useful information, because mage=0 is clearly outside the range of data.

It would be more meaningful to examine the effect of race for realistic ages of motherhood. For example, a baby born to a white mother who is the average age in the sample (29.5578603 years old) is predicted to have a birthweight that is (586.35235) + (-16.82123*29.5578603) = **89.153** <u>*grams more compared to nonwhite babies whose mothers are this same age*</u>.

You could also obtain this answer directly by re-defining terms in the model, and re-estimating the model. In particular, define the interaction term to equal zero when evaluated at the value of interest (in this case, the mean value of *mage*). [see full Stata program at back of notes]

```
*the following will set ave_mage=29.557 for all observations
egen ave_mage = mean(mage)

gen mwhmagenew = (mage-ave_mage)*mwhte

reg bwght mage mwhte mwhmagenew
```

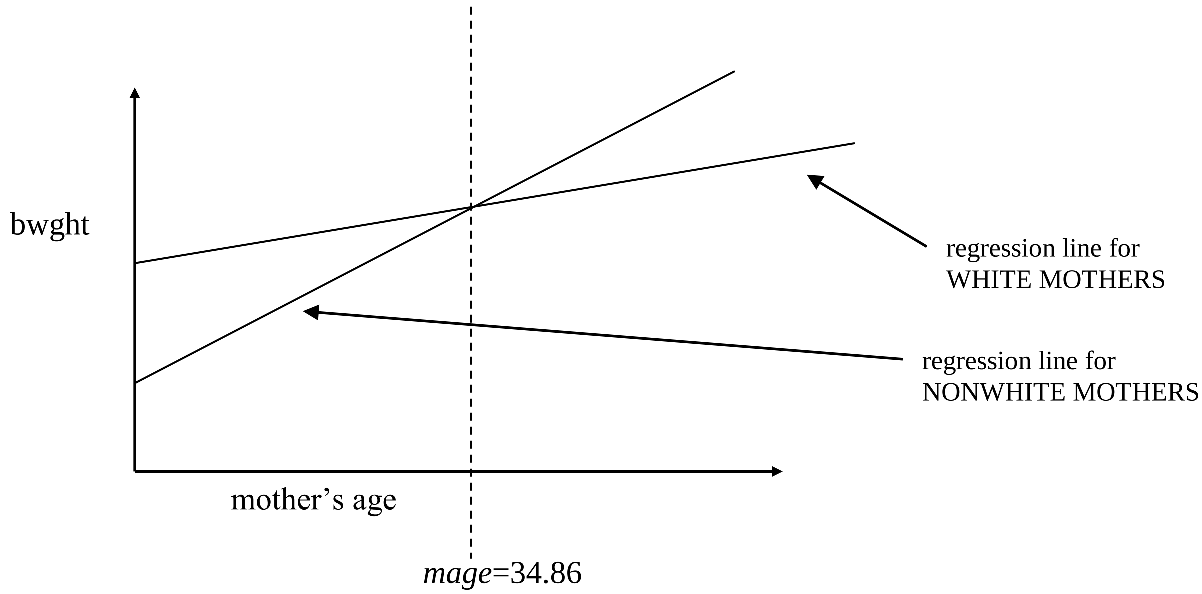
By doing this, you can look directly at the coefficient on *mwhte* to measure the difference between whites and nonwhites at the value of interest (e.g., average age of 29.557). This difference should be exactly equal to the prediction that we just calculated above, and we can look at the t-statistics on the *mwhte* coefficient to test its significance. Redefining the interaction term in this way just gives a convenient way to evaluate the effects of one term in the interaction when the other term in the interaction is equal to a particular value. If we wanted to evaluate the difference between whites compared to nonwhites at some other value of *mage*, we would need to either 1) redefine a new interaction term by subtracting off this new value of interest from *mage* in the interaction term and rerunning the model or 2) calculate that effect by hand using the original model.

The advantage of the first strategy (subtracting off the particular value of interest, e.g. the mean) is that it automatically calculates the relevant standard error and t-statistic (on the coefficient for *mwhte* in this case) and thus easily allows us to conduct hypothesis tests and calculate confidence intervals. THE STATA CODE AND OUTPUT AT END OF THIS SECTION (III) OF THE NOTES SHOWS HOW TO DO THIS. This method is also shown in example 6.3 on pp. 198-199 of Wooldridge, and is more generally introduced in Section 4.4 (pp. 140-143).

- At what point is the predicted birthweight for white and nonwhite mothers the same (i.e., at what Y do the regression lines cross?)

$$\hat{Y}_{white} = \hat{Y}_{nonwhite}$$
$$2773.68 + 18.58MAGE + 586.35 - 16.82MAGE = 2773.68 + 18.58MAGE =$$
$$586.35 - 16.82MAGE = 0$$
$$MAGE = 34.86$$

--This says that when moms are about 35 years old, we predict that birthweight of babies born to white and nonwhite moms will be the same (plug into the above equation, and that predicted birthweight is 3421.38 grams)



Example 2: (back to regression of birthweight on mother's age and race):

In a sense the following is a "fully interacted" model because there are only 2 independent variables and there is an interaction (mwhmage) between those variables included in the model.

```
. reg bwght mage mwhte mwhmage

      Source |       SS       df       MS              Number of obs =    1832
-------------+------------------------------           F(  3,  1828) =    3.67
       Model |  3646382.99     3     1215461           Prob > F      =  0.0118
    Residual |   604984392  1828  330954.262           R-squared     =  0.0060
-------------+------------------------------           Adj R-squared =  0.0044
       Total |   608630775  1831  332403.481           Root MSE      =  575.29

------------------------------------------------------------------------------
       bwght |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
        mage |   18.57864    7.92029     2.35   0.019     3.044867     34.1124
       mwhte |   586.3524   251.3175     2.33   0.020     93.45283    1079.252
     mwhmage |  -16.82123   8.475335    -1.98   0.047    -33.44359    -.198877
       _cons |   2773.682   234.4771    11.83   0.000     2313.811    3233.553
------------------------------------------------------------------------------
```

- Using the model above, how do you test the following question:

  Controlling for the mother's age, is average birthweight the same regardless of race? (i.e., holding *mage* constant, is there a race effect?; think of this as: does race matter at all?)

  We have to use an F-test to test this hypothesis:

  H_0: $\beta_{mwhte} = 0$ and $\beta_{mwhmage} = 0$;
  H_1: $\beta_{mwhte} \neq 0$ and/or $\beta_{mwhmage} \neq 0$;

```
. test mwhte = mwhmage = 0

 ( 1)  mwhte - mwhmage = 0
 ( 2)  mwhte = 0

       F(  2,  1828) =    4.47
            Prob > F =    0.0116
```

  - Note: Similarly, if we wanted to test whether age had no effect on birthweight *for mothers of the same race (i.e., holding race constant)*, we would need to test the joint null:

    H_0: $\beta_{mage} = 0 \; and \; \beta_{mwhmage} = 0$

What variables are in the RESTRICTED model? In the UNRESTRICTED model?

We reject the null at the 0.10 level, but not at the 0.05 level (F= 2.92, and p=0.0541).

IV. INTERACTIONS WITH TWO INTERVAL-RATIO VARIABLES

- Using the same kind of logic, we can interact two interval-ratio variables, e.g., mother's education (*meduc*) and father's education *(feduc)*. The interaction term *(mfe)* will tell us whether the effect of mother's education varies by level of father's education; or alternatively, whether the effect of father's education varies by mother's education.

```
. * create interval-ratio interaction
. gen mfe = meduc * feduc
(51 missing values generated)

. reg bwght mage meduc feduc mfe

      Source |       SS       df       MS              Number of obs =    1781
-------------+------------------------------           F(  4,  1776) =    2.15
       Model |  2807424.06     4  701856.015           Prob > F      =  0.0721
    Residual |   579096570  1776  326067.889           R-squared     =  0.0048
-------------+------------------------------           Adj R-squared =  0.0026
       Total |   581903994  1780  326912.356           Root MSE      =  571.02

------------------------------------------------------------------------------
       bwght |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
        mage |   1.694888   3.018027     0.56   0.574     -4.22437    7.614145
       meduc |   -57.9106   30.63663    -1.89   0.059    -117.9982    2.177041
       feduc |  -50.98418   29.53053    -1.73   0.084    -108.9024    6.934059
         mfe |   4.525061   2.182147     2.07   0.038     .2452137    8.804907
       _cons |   3979.003   401.2162     9.92   0.000     3192.098    4765.909
------------------------------------------------------------------------------
```

- The main effects are each statistically significant at the .10 level (p for *meduc*=.059), and the interaction is statistically significant (p=.038). The fact that the interaction is significant tells us that effects of mother's education varies by level of father's education; or alternatively, the effect of father's education varies by level of mother's education:

Computation of Effects at Selected Levels of Other Interacted Variable

	Effect of *feduc*	**Computation of feduc effect (at set level of *meduc*)**
meduc=8	-14.7837	= - 50.9842 + (4.52506*8)
meduc=12	3.31654	= - 50.9842 + (4.52506*12)
meduc=16	21.41678	= - 50.9842 + (4.52506*16)

	Effect of *meduc*	**Computation of meduc effect (at set level of *feduc*)**
feduc=8	-21.7101	= - 57.9106 + (4.52506*8)
feduc=12	-3.60988	= - 57.9106 + (4.52506*12)
feduc=16	14.49036	= - 57.9106 + (4.52506*16)

V. TESTING FOR DIFFERENCES ACROSS GROUPS IN REGRESSION FUNCTIONS (AKA, "Chow Test"): A FULLY INTERACTED MODEL

- In the examples above, we interacted only two variables; the slope of one variable was allowed to vary according to different values of another variable. What if we want **all** the coefficients in a model to be different according to different values of another variable?

- First, we'll talk about this in terms of two separate regression models; then we'll make the connection to a model that uses interactions, and show that these two different methods produce exactly the same results.

- Above, we looked at an example where only the slope of CIGS was allowed to vary, depending on whether the baby was male or female.

- What if we think that males and females have different coefficient estimates on ***all*** the other variables in the equation? If the effects of male and female babies essentially have different "processes" that predict *birthweight* from these variables, then we could think about estimating two separate regressions:

females: BWGHT = $\beta_0 + \beta_1$CIGS + β_2FAMINC + β_3MOTHEDUC + β_4FATHEDUC + η

males: BWGHT = $\alpha_0 + \alpha_1$CIGS + α_2FAMINC + α_3MOTHEDUC + α_4FATHEDUC + ω

- We'll test whether estimating these regression separately provides greater explanatory power than if we were to pool both females and males and estimate only one regression:

pooled: BWGHT = $\pi_0 + \pi_1$CIGS + π_2FAMINC + π_3MOTHEDUC + π_4FATHEDUC + ν

- This is known as a **Chow Test**, and the test statistic is:

$$F = \left(\frac{[SSR_{pooled} - (SSR_1 + SSR_2]}{SSR_1 + SSR_2}\right) * \left(\frac{n - 2(k+1)]}{k+1}\right)$$ with numerator d.f.= *k+1*

and denominator d.f. = *n-2(k+1)*

[note: think about why these are the d.f.!]

```
. bys male: reg bwght cigs faminc motheduc fatheduc

-------------------------------------------------------------------------------------
-> male = 0

      Source |       SS       df       MS              Number of obs =     573
-------------+------------------------------           F(  4,   568) =    5.27
       Model |  8757.85824     4  2189.46456           Prob > F      =  0.0004
    Residual |  235977.779   568  415.453836           R-squared     =  0.0358
-------------+------------------------------           Adj R-squared =  0.0290
       Total |  244735.637   572  427.859505           Root MSE      =  20.383

------------------------------------------------------------------------------
       bwght |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
        cigs |  -.5858689   .1574105    -3.72   0.000    -.8950466   -.2766912
      faminc |   .0727672   .0551728     1.32   0.188    -.0356005    .1811349
    motheduc |  -.9634324   .4832006    -1.99   0.047     -1.91251   -.0143543
    fatheduc |     .66321   .4149273     1.60   0.111    -.1517691    1.478189
       _cons |   120.2174   4.901743    24.53   0.000     110.5897    129.8452
------------------------------------------------------------------------------

-------------------------------------------------------------------------------------
-> male = 1

      Source |       SS       df       MS              Number of obs =     618
-------------+------------------------------           F(  4,   613) =    5.31
       Model |  7828.3217     4  1957.08042           Prob > F      =  0.0003
    Residual |  225981.997   613  368.649261           R-squared     =  0.0335
-------------+------------------------------           Adj R-squared =  0.0272
       Total |  233810.319   617  378.947032           Root MSE      =    19.2

------------------------------------------------------------------------------
       bwght |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
        cigs |  -.5485773   .1553238    -3.53   0.000    -.8536086   -.2435461
      faminc |   .0515484   .0487796     1.06   0.291    -.0442471    .1473438
    motheduc |   .0277863   .4241947     0.07   0.948     -.805265    .8608375
    fatheduc |   .3554441   .3876648     0.92   0.360     -.405868    1.116756
       _cons |    115.544   5.001263    23.10   0.000     105.7223    125.3656
------------------------------------------------------------------------------

* pooled regression with males & females
. reg bwght cigs faminc motheduc fatheduc

      Source |       SS       df       MS              Number of obs =    1191
-------------+------------------------------           F(  4,  1186) =   10.05
       Model |  15827.6593     4  3956.91482           Prob > F      =  0.0000
    Residual |  466919.033  1186   393.69227           R-squared     =  0.0328
-------------+------------------------------           Adj R-squared =  0.0295
       Total |  482746.692  1190  405.669489           Root MSE      =  19.842

------------------------------------------------------------------------------
       bwght |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
        cigs |  -.5894954   .1106172    -5.33   0.000    -.8065225   -.3724682
      faminc |   .0538254   .0366502     1.47   0.142    -.0180811    .1257319
    motheduc |  -.4379234   .3197377    -1.37   0.171    -1.065238    .1893912
    fatheduc |   .4936695   .2832896     1.74   0.082    -.0621351    1.049474
       _cons |   118.0741   3.500291    33.73   0.000     111.2066    124.9415
```

- Calculate the F-statistic:

$$F = \left(\frac{[SSR_{pooled} - (SSR_1 + SSR_2]}{SSR_1 + SSR_2}\right) * \left(\frac{n - 2(k+1)]}{k+1}\right)$$

$$F = \left(\frac{[466{,}919 - (235{,}978 + 225{,}982]}{235{,}978 + 225{,}982}\right) * \left(\frac{1{,}191 - 2(4+1)]}{4+1}\right)$$

$$F = \left(\frac{4{,}959}{461{,}960}\right) * \left(\frac{1{,}181}{5}\right) = 2.5355$$

F-critical = (5, 1181) at alpha=0.05: 2.21; at alpha=0.01: 3.02. Therefore, reject the null at the 0.05 significance level; fail to reject at the 0.01 level.

- This test doesn't allow us to know WHICH coefficient(s) are different across the two models (this is no different from what we could learn from earlier F-tests).

ALTERNATIVE, USING INTERACTIONS:

- The same kind of test can be achieved using interaction terms. When **all** variables in a model are interacted with one of the indicator variables, it is referred to as a **fully-interacted model**, and it is equivalent to estimating two separate models.

BWGHT = δ_0 + δ_1CIGS + δ_2FAMINC + δ_3MOTHEDUC + δ_4FATHEDUC +

+ δ_5MALE + δ_6MCIGS + δ_7MFAMINC + δ_8MMOTHED + δ_9MFATHED + φ

- Does gender matter in explaining birthweight? The null to be tested using the fully-interacted model is:

H0: $\delta_5=0$ and $\delta_6=0$ and $\delta_7=0$ and $\delta_8=0$ and $\delta_9=0$
H1: at least one of these coefficients is different from zero

- To test the hypothesis above, we can just use an F-test that we already know. Because we are testing all the coefficients that are interacted with *male*, this is also a **Chow test:**

```
.
. reg bwght cigs faminc motheduc fatheduc male mcigs mfaminc mmothed mfathed

      Source |       SS       df       MS              Number of obs =    1191
-------------+------------------------------           F(  9,  1181) =    5.90
       Model |   20786.916     9  2309.65734           Prob > F      =  0.0000
    Residual |  461959.776  1181  391.159844           R-squared     =  0.0431
-------------+------------------------------           Adj R-squared =  0.0358
       Total |  482746.692  1190  405.669489           Root MSE      =  19.778



------------------------------------------------------------------------------
       bwght |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
        cigs |  -.5858689   .1527388    -3.84   0.000    -.8855386   -.2861992
      faminc |   .0727672   .0535354     1.36   0.174    -.0322679    .1778023
    motheduc |  -.9634324     .46886    -2.05   0.040    -1.883324   -.0435409
    fatheduc |     .66321   .4026129     1.65   0.100    -.1267064    1.453126
        male |  -4.673477   7.011565    -0.67   0.505    -18.42999    9.083036
       mcigs |   .0372915   .2211962     0.17   0.866    -.3966899     .471273
     mfaminc |  -.0212188    .073422    -0.29   0.773    -.1652709    .1228332
     mmothed |   .9912187   .6409045     1.55   0.122    -.2662197    2.248657
     mfathed |  -.3077659   .5670607    -0.54   0.587    -1.420325    .8047928
       _cons |   120.2174   4.756268    25.28   0.000     110.8858    129.5491
------------------------------------------------------------------------------

. * test male and all interactions equal to zero
. test male = mcigs = mfaminc = mmothed = mfathed = 0

 ( 1)  male - mcigs = 0
 ( 2)  male - mfaminc = 0
 ( 3)  male - mmothed = 0
 ( 4)  male - mfathed = 0
 ( 5)  male = 0

       F(  5,  1181) =    2.54
            Prob > F =    0.0271
```

- What variables are in the RESTRICTED model?
  CIGS, MOTHEDUC, FATHEDUC, FAMINC

  In the UNRESTRICTED model? CIGS, FAMINC, MOTHEDUC, FATHEDUC, MALE, MCIGS, MFAMINC, MMOTHEDUC, MFATHEDU,

- We reject the null that all the coefficients are equal to zero: we conclude that it's likely that the regression functions **do** differ for males and females.

  Happily, this is the same F-stat (and conclusion) that Stata calculated when we ran the Chow test using separate regressions and a pooled regression.

- Note that the null hypothesis we test doesn't have to be all-or-nothing. We can test subsets of the interacted terms, using the F-test we already know.

- For example, another null of interest is: Do the effects of CIGS, FAMINC, MOTHEDUC, and FATHEDUC on *birthweight* vary by gender? (i.e., still allow for a difference in intercept).

  H0: δ_6=0 and δ_7=0 and δ_8=0 and δ_9=0
  H1: at least one of these coefficients is different from zero

```
. * test male interactions equal to zero (not including male itself)
. test mcigs = mfaminc = mmothed = mfathed = 0

 ( 1)  mcigs - mfaminc = 0
 ( 2)  mcigs - mmothed = 0
 ( 3)  mcigs - mfathed = 0
 ( 4)  mcigs = 0

       F(  4,  1181) =    0.64
            Prob > F =    0.6314
```

```
/***************************************************************************
PPOL 503
Course Notes 3: Interactions
***************************************************************************/
cd "C:\...\PPOL509\Stata datasets"

capture: log close
log using "..\do files\notes14.txt", text replace
set more off

/***************************************************************************
Race and Education: bwght.dta data set
***************************************************************************/
clear
use bwght.dta

* Create Interaction Variables
gen female    = 1-male
gen whmale    = white*male
gen whfemale  = white*female
gen nonwh     = 1-white
gen nonwhmale = nonwh*male
gen mcigs     = male*cigs
gen mfaminc   = male*faminc
gen mmothed   = male*motheduc
gen mfathed   = male*fatheduc

*** Interactions with two indicator variables
reg bwght white male
reg bwght white male whmale
reg bwght whfemale nonwhmale whmale

* Statistics by gender and race
bys male white: summarize bwght

*** Fully Interacted Model: Chow Test
bys male: reg bwght cigs faminc motheduc fatheduc
* pooled regression with males & females
reg bwght cigs faminc motheduc fatheduc

reg bwght cigs faminc motheduc fatheduc male mcigs mfaminc mmothed mfathed
* test male and all interactions equal to zero
test male = mcigs = mfaminc = mmothed = mfathed = 0
* test male interactions equal to zero (not including male itself)
test mcigs = mfaminc = mmothed = mfathed = 0


/***************************************************************************
Mother's Age: bwght2.dta data set
***************************************************************************/
clear
use bwght2.dta

*** Interactions with one indicator and one interval-ratio variable
gen mwhmage = mwhte*mage
```

```
reg bwght mage mwhte mwhmage
test mwhte = mwhmage = 0
test mage = mwhmage = 0

* Effect of whmale at average value of Mage
egen ave_mage = mean(mage)     // create new variable with mean age
sum ave_mage                   // check value of mean
gen mwhmagenew = (mage-ave_mage)*mwhte
list bwght mage mwhte mwhmage ave_mage mwhmagenew in 1/10
reg bwght mage mwhte mwhmagenew


*** Interactions with two interval-ratio variables
* create interval-ratio interaction
gen mfe = meduc * feduc
reg bwght mage meduc feduc mfe

log close
```

STATA CODE TO PRODUCE EFFECT OF *MWHTE* AT AVERAGE *MAGE*:

```
* Effect of whmale at average value of Mage
egen ave_mage = mean(mage)     // create new variable with mean age
sum ave_mage                   // check value of mean
gen mwhmagenew = (mage-ave_mage)*mwhte
list bwght mage mwhte mwhmage mmage ave_mage mwhmagenew in 1/10
reg bwght mage mwhte mwhmagenew
```

STATA OUTPUT:

```
. sum ave_mage

    Variable |       Obs        Mean    Std. Dev.       Min        Max
-------------+--------------------------------------------------------
    ave_mage |      1832    29.55786           0   29.55786   29.55786


. list bwght mage mwhte mwhmage ave_mage mwhmagenew in 1/10

     +---------------------------------------------------+
     | bwght   mage   mwhte   mwhmage   ave_mage   mwhmage~w |
     |---------------------------------------------------|
  1. |  3060     26       0         0   29.55786           0 |
  2. |  3730     29       1        29   29.55786   -.5578594 |
  3. |  2530     33       1        33   29.55786    3.442141 |
  4. |  3289     28       1        28   29.55786   -1.557859 |
  5. |  3590     23       1        23   29.55786   -6.557859 |
     |---------------------------------------------------|
  6. |  3420     28       1        28   29.55786   -1.557859 |
  7. |  3355     27       1        27   29.55786   -2.557859 |
  8. |  3459     41       0         0   29.55786           0 |
  9. |  3590     32       1        32   29.55786    2.442141 |
 10. |  4410     16       1        16   29.55786   -13.55786 |
     +---------------------------------------------------+
```

```
. reg bwght mage mwhte mwhmagenew

      Source |       SS       df       MS              Number of obs =    1832
-------------+------------------------------           F(  3,  1828) =    3.67
       Model |  3646382.99     3     1215461           Prob > F      =  0.0118
    Residual |   604984392  1828  330954.262           R-squared     =  0.0060
-------------+------------------------------           Adj R-squared =  0.0044
       Total |   608630775  1831  332403.481           Root MSE      =  575.29

------------------------------------------------------------------------------
       bwght |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
        mage |   18.57864    7.92029     2.35   0.019     3.044867     34.1124
       mwhte |   89.15266   42.47618     2.10   0.036     5.845718    172.4596
  mwhmagenew |  -16.82123   8.475335    -1.98   0.047    -33.44359    -.198877
       _cons |   2773.682   234.4771    11.83   0.000     2313.811    3233.553
------------------------------------------------------------------------------
```

NOTE: Notice that all the coefficients, t-statistics, F-statistic, R-squared, etc. values in this model are exactly the same as those in the original model *except for* the coefficient and t-state on *mwhte*.. For the reasons stated above, the coefficient on *mwhte* now gives you the effect of the mother being white (compared to being nonwhite) on the baby's birthweight *when* ***mage*** *is equal to its mean value in the sample*. The coefficient estimate of $\hat{\beta}_{mwhte}$ = 89.153, exactly what we calculated by hand above from the regression estimated without subtracting 29.557 from *mage* when creating the interaction term *mwhmagenew*.