PPOL502-01, Spring 2016
Course Notes #4: Further Understanding of "Holding Constant" and "Partialling Out" in Multivariate Regression

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I. WHAT DOES IT MEAN TO "HOLD OTHER FACTORS CONSTANT"?

- The following comments refer to the additional pages of regression output and graphs with the created variables POLICE, CRIME, and CRIMINALS. This example builds on and expands an example created by Erzo F.P. Luttmer while at the Univ. of Chicago.

- First, we'll walk through the Stata **list**, **summarize**, and **correllate** output to get a sense of the data….
  ** CRIME1 through CRIME4 are four different dependent variables. POLICE and CRIMINALS are the independent variables of interest.
  ** Note: the correlation matrix shows that the correlation between CRIMINALS and every one of the CRIME# variables is exactly 0.94516. Why?

- *Regression #1* series:

  -- Including only POLICE in the regression to explain the number of crimes (CRIME1), we find that there is a (stat sig) positive relationship between the number of police and the number of crimes: for each additional police person, we predict that crimes will increase by 0.79 (or a little less than one). See Regression 1-A.

  -- Figure 1-A, along with the printout of the raw data, shows something curious though: we see three clusters of 3 observations in each cluster. Within each cluster, when the number of police is low, so are crimes, and *so are the number of criminals.*

  -- If we want to know the relationship between the number of police and the number of crimes, holding the number of criminals constant, then, we need to incorporate this extra information into the regression. Regression 1-B does this (note: there is no regression plot shown for this regression: why not? it is in a higher dimension and can't be represented on these two-dimensional graphs).

  Now, holding the number of criminals constant, for each additional police person, regression 1-B indicates that we predict that the number of crimes will decrease by one; there is a one-to-one relationship between police and crimes, holding the number of criminals constant.

  -- Even though we couldn't plot Regression 1-B, we can get a sense for what's going on by an auxiliary set of plots. NOTE: you wouldn't run such a set of plots in "real life." We show them here to give you a sense of the concept "holding constant." See Figures 1-C, 1-D, and 1-E.

  -- How do we interpret the coefficient on CRIMINAL in Regression 1-B?

-- Also note the correlation matrix and compare with the printout of the data (p. 1 of output).

-- We'll return to these examples in the discussion of omitted variables in a later set of Course Notes.

- *Regression #2 series*

You may think that this example is rigged (it is). Nothing could be this perfect in the real world, you say (it's not). But this can help to develop your intuition for what "holding other things constant" means. Let's change the example a little bit:

-- what happens when the relationship between police and crimes, holding criminals constant, is not so perfect (or at least not perfect in this way)? – we'll now use CRIME2 as the dependent variable

-- The relationship between POLICE and CRIME2 is negative when the number of CRIMINALS is equal to 2 or to 5; but *positive* when the number of criminals is equal to 8. See the printout, and Figure 2-A and Figures 2-C through 2-E.

-- Given this info, compared to the 0.79 coefficient on POLICE on CRIME1 in Regression 1-A (i.e., *not* controlling for the number of criminals?), what do you expect will happen to the coefficient on POLICE on CRIME2? Now look at the regression estimates for 2-A.

-- Given this info, compared to the –1.01 coefficient on POLICE in Regression 1-B (i.e., controlling for the number of criminals), what do you think the relationship between POLICE and CRIME2 will be, controlling for the number of criminals? Now look at the regression estimates for 2-B.

- *Regression series #3, #4*

-- We can continue to change the relationships between police and crimes, redefining the CRIME dependent variable as CRIME3 and CRIME4; and examine the regression lines estimated.

	Bivariate Regression				*Multiple Regression (holding CRIMINALS constant)*			
DepVar	**POLICE Coeff**	**s.e.**	**p-value**	**R-squared**	**POLICE Coeff**	**s.e.**	**p-value**	**R-squared**
CRIME1	0.79335	0.22877	0.0104	0.6321	-1.01409	0.07508	<0.0001	0.9966
CRIME2	0.86110	0.19059	0.0027	0.7446	-0.33658	0.39768	0.4298	0.9047
CRIME3	0.93776	0.12893	0.0002	0.8831	0.43004	0.38237	0.3037	0.9119
CRIME4	0.99617	0.02210	<0.0001	0.9966	1.01409	0.07508	<0.0001	0.9966

II. WHAT DOES THE "PARTIALLING-OUT" INTERPRETATION MEAN?

- Wooldridge talks about the partialling out interpretation. What in the world is he talking about?

- Recall the regression output from 1-B (using *CRIME1* as depvar):

crime1	Coef.	Std. Err.	t	P>|t|	[95% Conf. Interval]	
criminal	2.008263	.0791434	25.37	0.000	1.814606	2.20192
police	-1.014086	.075082	-13.51	0.000	-1.197805	-.8303668
_cons	-.0192298	.1393464	-0.14	0.895	-.3601981	.3217384

- To understand the "partialling out" interpretation of regression coefficients, we use residuals from regressions of the X_1 (the variable of interest) on the other *Xs* in the model. It's easiest to show by example.

- To get predicted values from a regression, you can use the following type of code:

```
regress police criminal
predict polhat , xb
predict polresid , residuals
```

NOTE: The parameter estimates from this regression are not interesting in and of themselves; we are running this regression to obtain the residuals:

```
. regress police criminal

      Source |       SS       df       MS              Number of obs =       9
-------------+------------------------------           F(  1,     7) =   63.00
       Model |         54     1          54            Prob > F      =  0.0001
    Residual |          6     7  .857142857            R-squared     =  0.9000
-------------+------------------------------           Adj R-squared =  0.8857
       Total |         60     8         7.5            Root MSE      =  .92582
```

police	Coef.	Std. Err.	t	P>|t|	[95% Conf. Interval]	
criminal	1	.1259882	7.94	0.000	.7020853	1.297915
_cons	8.88e-16	.7014724	0.00	1.000	-1.658719	1.658719

```
. list police criminal polhat polresid, clean

     police   criminal   polhat   polresid
  1.      1          2        2         -1
  2.      2          2        2          0
  3.      3          2        2          1
  4.      4          5        5         -1
  5.      5          5        5          0
  6.      6          5        5          1
  7.      7          8        8         -1
  8.      8          8        8          0
  9.      9          8        8          1
```

- *The residuals from the above regression are the parts of X_1 (POLICE) that are <u>un</u>correlated with X_2 (CRIMINAL)*

- So, if we regress the original dependent variable, *CRIME1*, on these residuals, what happens?:

```
. regress crime1 polresid

      Source |       SS       df       MS              Number of obs =       9
-------------+------------------------------           F(  1,     7) =    0.81
       Model |  6.1702209     1  6.1702209             Prob > F      =  0.3991
    Residual |  53.575892     7  7.65369886            R-squared     =  0.1033
-------------+------------------------------           Adj R-squared = -0.0248
       Total |  59.7461129     8  7.46826412           Root MSE      =  2.7665

------------------------------------------------------------------------------
      crime1 |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
    polresid |  -1.014086   1.129432    -0.90   0.399    -3.684768    1.656596
       _cons |   4.951655   .9221773     5.37   0.001     2.771052    7.132258
------------------------------------------------------------------------------
```

- This shows the relationship between *POLICE* and *CRIME1*, <u>after the influence of *CRIMINALS* has been netted out</u>.

- Note that the coefficient estimate on *POLRESID* is exactly the same as the coefficient estimate on *POLICE* in the multiple regression model above.

- Note that the standard errors (and thus *t*-values and *p*-values) from the one-step method and the two-step method were different, even though the coefficient estimates were the same:

- The s.e.'s in the (one-step) equation are the <u>correct</u> ones. While the coefficient on *POLRESID* in the second equation is <u>not biased, it is inefficient</u>. We'll talk more about that in later Course Notes.

```
cd "C:\Documents and Settings\mdb96\My Documents\sa\lectures\PPOL509\Stata datasets"

capture: log close
log using notes4.txt, text replace
set more off

clear
use crime.dta


/*****************************************************************************
Predicted Values Example: Notes pg. 3
*****************************************************************************/

regress police criminal
predict polhat , xb
predict polresid , residuals

list police criminal polhat polresid, clean

regress crime1 polresid


/*****************************************************************************
Summary Statistics
*****************************************************************************/

list crime1-crime4 police criminal
summarize crime1-crime4 police criminal
correlate crime1-crime4 police criminal

/*****************************************************************************
Regression #1 Series
*****************************************************************************/

* Regression 1-A
regress crime1 police
* Regression 1-B
regress crime1 criminal police
* Regression 1-C
regress crime1 police if criminal == 2
* Regression 1-D
regress crime1 police if criminal == 5
* Regression 1-E
regress crime1 police if criminal == 8

/*****************************************************************************
Regression #2 Series
*****************************************************************************/

* Regression 2-A
regress crime2 police
* Regression 2-B
regress crime2 criminal police
* Regression 2-C
regress crime2 police if criminal == 2
* Regression 2-D
regress crime2 police if criminal == 5
* Regression 2-E
regress crime2 police if criminal == 8

/*****************************************************************************
Regression #3 Series
```

```
*****************************************************************************/

* Regression 3-A
regress crime3 police
* Regression 3-B
regress crime3 criminal police
* Regression 3-C
regress crime3 police if criminal == 2
* Regression 3-D
regress crime3 police if criminal == 5
* Regression 3-E
regress crime3 police if criminal == 8

/*****************************************************************************
Regression #4 Series
*****************************************************************************/

* Regression 4-A
regress crime4 police
* Regression 4-B
regress crime4 criminal police
* Regression 4-C
regress crime4 police if criminal == 2
* Regression 4-D
regress crime4 police if criminal == 5
* Regression 4-E
regress crime4 police if criminal == 8


/*****************************************************************************
Graphs
*****************************************************************************/

/*****************************************************************************
A - Series
*****************************************************************************/
* Regression 1-A
graph twoway (scatter crime1 police) (lfit crime1 police, range(0 10)) ///
       , xscale(range(0 10)) xlabel(0(1)10) xtitle("Police") ///
         yscale(range(0 10)) ylabel(0(1)10) ytitle("Crime1") ///
         legend(off)
graph export 1-A.wmf, replace

* Regression 2-A
graph twoway (scatter crime2 police) (lfit crime1 police, range(0 10)) ///
       , xscale(range(0 10)) xlabel(0(1)10) xtitle("Police") ///
         yscale(range(0 10)) ylabel(0(1)10) ytitle("Crime2") ///
         legend(off)
graph export 2-A.wmf, replace

* Regression 3-A
graph twoway (scatter crime3 police) (lfit crime1 police, range(0 10)) ///
       , xscale(range(0 10)) xlabel(0(1)10) xtitle("Police") ///
         yscale(range(0 10)) ylabel(0(1)10) ytitle("Crime3") ///
         legend(off)
graph export 3-A.wmf, replace

* Regression 4-A
graph twoway (scatter crime4 police) (lfit crime1 police, range(0 10)) ///
       , xscale(range(0 10)) xlabel(0(1)10) xtitle("Police") ///
         yscale(range(0 10)) ylabel(0(1)10) ytitle("Crime4") ///
         legend(off)
graph export 4-A.wmf, replace
```

```
/******************************************************************************
1-C through 1-E
******************************************************************************/
* Regression 1-C
graph twoway (scatter crime1 police) (lfit crime1 police, range(0 4)) ///
              if criminal == 2 , ///
              xscale(range(0 10)) xlabel(0(1)10) xtitle("Police") ///
              yscale(range(0 10)) ylabel(0(1)10) ytitle("Crime1") ///
              legend(off) title("Criminal = 2")
graph export 1-C.wmf, replace

* Regression 1-D
graph twoway (scatter crime1 police) (lfit crime1 police, range(0.75 9.25)) ///
              if criminal == 5 , ///
              xscale(range(0 10)) xlabel(0(1)10) xtitle("Police") ///
              yscale(range(0 10)) ylabel(0(1)10) ytitle("Crime1") ///
              legend(off) title("Criminal = 5")
graph export 1-D.wmf, replace

* Regression 1-E
graph twoway (scatter crime1 police) (lfit crime1 police, range(6 10)) ///
              if criminal == 8 , ///
              xscale(range(0 10)) xlabel(0(1)10) xtitle("Police") ///
              yscale(range(0 10)) ylabel(0(1)10) ytitle("Crime1") ///
              legend(off) title("Criminal = 8")
graph export 1-E.wmf, replace


/******************************************************************************
2-C through 2-E
******************************************************************************/
* Regression 2-C
graph twoway (scatter crime2 police) (lfit crime2 police, range(0 4)) ///
              if criminal == 2 , ///
              xscale(range(0 10)) xlabel(0(1)10) xtitle("Police") ///
              yscale(range(0 10)) ylabel(0(1)10) ytitle("Crime2") ///
              legend(off) title("Criminal = 2")
graph export 2-C.wmf, replace

* Regression 2-D
graph twoway (scatter crime2 police) (lfit crime2 police, range(0.75 9.25)) ///
              if criminal == 5 , ///
              xscale(range(0 10)) xlabel(0(1)10) xtitle("Police") ///
              yscale(range(0 10)) ylabel(0(1)10) ytitle("Crime2") ///
              legend(off) title("Criminal = 5")
graph export 2-D.wmf, replace

* Regression 2-E
graph twoway (scatter crime2 police) (lfit crime2 police, range(.25 10)) ///
              if criminal == 8 , ///
              xscale(range(0 10)) xlabel(0(1)10) xtitle("Police") ///
              yscale(range(0 10)) ylabel(0(1)10) ytitle("Crime2") ///
              legend(off) title("Criminal = 8")
graph export 2-E.wmf, replace
```

```
/*****************************************************************************
3-C through 3-E
*****************************************************************************/
* Regression 3-C
graph twoway (scatter crime3 police) (lfit crime3 police, range(0 4)) ///
              if criminal == 2 , ///
              xscale(range(0 10)) xlabel(0(1)10) xtitle("Police") ///
              yscale(range(0 10)) ylabel(0(1)10) ytitle("Crime3") ///
              legend(off) title("Criminal = 2")
graph export 3-C.wmf, replace

* Regression 3-D
graph twoway (scatter crime3 police) (lfit crime3 police, range(0.75 9.25)) ///
              if criminal == 5 , ///
              xscale(range(0 10)) xlabel(0(1)10) xtitle("Police") ///
              yscale(range(0 10)) ylabel(0(1)10) ytitle("Crime3") ///
              legend(off) title("Criminal = 5")
graph export 3-D.wmf, replace

* Regression 3-E
graph twoway (scatter crime3 police) (lfit crime3 police, range(.25 10)) ///
              if criminal == 8 , ///
              xscale(range(0 10)) xlabel(0(1)10) xtitle("Police") ///
              yscale(range(0 10)) ylabel(0(1)10) ytitle("Crime3") ///
              legend(off) title("Criminal = 8")
graph export 3-E.wmf, replace


/*****************************************************************************
4-C through 4-E
*****************************************************************************/
* Regression 4-C
graph twoway (scatter crime4 police) (lfit crime4 police, range(0 10)) ///
              if criminal == 2 , ///
              xscale(range(0 10)) xlabel(0(1)10) xtitle("Police") ///
              yscale(range(0 10)) ylabel(0(1)10) ytitle("Crime4") ///
              legend(off) title("Criminal = 2")
graph export 4-C.wmf, replace

* Regression 4-D
graph twoway (scatter crime4 police) (lfit crime4 police, range(0.75 9.25)) ///
              if criminal == 5 , ///
              xscale(range(0 10)) xlabel(0(1)10) xtitle("Police") ///
              yscale(range(0 10)) ylabel(0(1)10) ytitle("Crime4") ///
              legend(off) title("Criminal = 5")
graph export 4-D.wmf, replace

* Regression 4-E
graph twoway (scatter crime4 police) (lfit crime4 police, range(.25 10)) ///
              if criminal == 8 , ///
              xscale(range(0 10)) xlabel(0(1)10) xtitle("Police") ///
              yscale(range(0 10)) ylabel(0(1)10) ytitle("Crime4") ///
              legend(off) title("Criminal = 8")
graph export 4-E.wmf, replace

log close
```