PPOL 503-03, PPOL 503-04, Fall 2016
Course Notes #7 Ordered Probit and Ordered Logit Estimation

Dependent variable takes on values that are irrelevant (categories), but larger values correspond to “higher” outcomes. The outcomes can be ordered.

Examples: health status (poor, good, excellent); quality of service (low, medium, high); results of taste tests; employment status (unemployed, part time or full time).

Ordered Probit

$$Y^* = \beta_i X_i + u$$

Y* is unobserved. Rather we observe the following:

$Y = 0$ if $Y^* \le 0$,
$Y = 1$ if $0 < Y^* \le \mu_1$,
$Y = 2$ if $\mu_1, < Y^* \le \mu_2$,
…
…
$Y = J$ if $\mu_{J-1} \le Y^*$

An underlying score for individual-i is a linear function of X’s and cut-off points. Predicted probabilities corresponding to each category, for each observation, can be obtained. Since the probit is based on the normal distribution, we have the following probabilities:

$\text{Prob}\ (Y = 0) = \Phi\ (- \beta_i X_i)$.
$\text{Prob}\ (Y = 1) = \Phi\ (\mu_1 - \beta_i X_i) - \Phi\ (- \beta_i X_i)$.
$\text{Prob}\ (Y = 2) = \Phi\ (\mu_2 - \beta_i X_i) - \Phi\ (\mu_1 - \beta_i X_i)$
…
…
$\text{Prob}\ (Y = J) = 1 - \Phi\ (\mu_{J-1} - \beta_i X_i)$

For all the probabilities to be positive it must be true that:

$$0 < \mu_1 < \mu_2 < \ldots. \quad \ldots. < \mu_{J-1}$$

The μ’s are unknown parameters to be estimated with β_i. Consider for example an opinion survey. The respondents have their own intensity of feelings, which depends on measureable factors X and unobservable factors u. Each person could respond to their own Y* if asked to do so. Given only five choices, each selects the category that most closely represents his/her own feelings.

The probability of observing outcome- j is the probability that the estimated linear function plus an error is within the range of cut-off points estimated for the outcome- j. U_i is logistically distributed in ordered logit; normally distributed for ordered probit.

Obtain estimates of β and cut-off points with MLE.
As is the case with binary probit the estimate of β indicates the significance and direction of the effect of each X variable on the outcome of interest but it does not reveal the magnitude of the effect. Need to calculate the marginal effects to assess the relative importance of each X variable on the outcome of interest.
If there are M categories in the dependent variable, there will be M-1 cut-off points estimated.

One of the assumptions underlying ordered probit (logit) regression is that the relationship between each pair of outcome groups is the same. In other words, ordered probit (logit) regression assumes that the coefficients that describe the relationship between, say, the lowest versus all higher categories of the response variable are the same as those that describe the relationship between the next lowest category and all higher categories, etc. This is called the proportional odds assumption or the parallel regression assumption. Because the relationship between all pairs of groups is the same, there is only one set of coefficients (only one model). If this was not the case, we would need different models to describe the relationship between each pair of outcome groups.

NOTE: This notation is from Greene (1993). This is confusing because Greene includes an intercept in his $\beta_i X_i$ term and Stata output does not. So Greene writes the Prob (Y = 0) = Φ (- $\beta_i X_i$).

In the Stata example below cut1 is Greene's intercept but with a reversed sign, so with Stata output the probit index $\beta_i X_i$ does not include an intercept. Therefore when Stata calculates the predicted probabilities for each outcome it uses the cut points and the parameters from the ordered probit model.

EXAMPLE: Ordered Probit Model Predicting Frequency of Use of Physical Therapy at School During Past 6 Months

Dependent Variable: Ordinal Variable =0 if child received no physical therapy at school during last 6 months; = 1 if child received infrequent physical therapy at school during last 6 months; = 2 if child received regular in the past 6 months (2 times per week to once per month); = 3 if child received frequent physical therapy (more than once per week).
Interpretation of Results:

1. Contrary to the multinomial logit model, the results yield one equation predicting the outcome of interest.

2. A negative coefficient means the effect of this variable moves the special needs child in the direction of being a non-user of physical therapy services.
3. A positive coefficient means the effect of this variable moves the child in the direction of reporting being a frequent user of physical therapy services.

4. Probit coefficients only indicate the direction and significance of changing an individual regressor on the outcome of interest. Need to calculate marginal impacts to determine magnitude of the effects.

```
oprobit schphysther ffs $regress

Iteration 0:   log likelihood =  -701.4939
Iteration 1:   log likelihood =  -636.7239
Iteration 2:   log likelihood = -635.27554
Iteration 3:   log likelihood = -635.26947
Iteration 4:   log likelihood = -635.26947

Ordered probit regression                         Number of obs   =        830
                                                  LR chi2(24)     =     132.45
                                                  Prob > chi2     =     0.0000
Log likelihood = -635.26947                       Pseudo R2       =     0.0944

------------------------------------------------------------------------------
 schphysther |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
         ffs |  -.3154582    .102016    -3.09   0.002    -.5154058   -.1155105
        good |   .1658189   .1107088     1.50   0.134    -.0511664    .3828041
    fairpoor |    .052398   .1351755     0.39   0.698     -.212541    .3173371
main_chronic |  -.2900373   .1339497    -2.17   0.030    -.5525739   -.0275007
main_therapy |  -.1990542   .1424614    -1.40   0.162    -.4782735     .080165
  main_birth |   .1649923   .1386534     1.19   0.234    -.1067634     .436748
   main_none |  -.4879172   .2783575    -1.75   0.080    -1.033488    .0576535
   comorbid1 |   .2745891   .1974297     1.39   0.164     -.112366    .6615441
   comorbid2 |   .5253991   .1923165     2.73   0.006     .1484657    .9023324
   comorbid3 |   .7256254     .18576     3.91   0.000     .3615425    1.089708
   comorbid4 |   .7811432   .1922341     4.06   0.000     .4043713    1.157915
   comorbid5 |   1.139567   .2118224     5.38   0.000     .7244025    1.554731
 comorbid6up |   1.201113   .1998404     6.01   0.000     .8094331    1.592793
       ageg1 |   .4944938   .1887733     2.62   0.009     .1245048    .8644827
       ageg2 |   .5604288   .1403904     3.99   0.000     .2852686     .835589
       ageg3 |   .4300658   .1410113     3.05   0.002     .1536887     .706443
        pars |   .0053662   .0040392     1.33   0.184    -.0025506    .0132829
     numadl3 |   .0190009   .0354442     0.54   0.592    -.0504686    .0884703
    youngmom |   .0229518   .1309251     0.18   0.861    -.2336568    .2795604
     incomep |  -.0000679   .0001368    -0.50   0.620     -.000336    .0002002
      hsgrad |  -.1354445   .1153874    -1.17   0.240    -.3615996    .0907106
    somecoll |  -.2896036   .1780095    -1.63   0.104    -.6384959    .0592886
 collegeplus |  -.0165108   .3366939    -0.05   0.961    -.6764188    .6433972
        ced2 |   .0002874   .0040204     0.07   0.943    -.0075925    .0081673
-------------+----------------------------------------------------------------
       /cut1 |   1.745851   .4314071                      .9003086    2.591393
       /cut2 |   1.830795   .4316553                      .9847659    2.676824
       /cut3 |    2.25881   .4336105                      1.408949     3.10867

In this example cut1 = 1.746, cut2 = 1.831 and cut3 = 2.259.
```

```
.mfx compute, predict (outcome (0))

Marginal effects after oprobit
      y  = Pr(schphysther==0) (predict, outcome (0))
         =  .7529633
------------------------------------------------------------------------------
variable |      dy/dx    Std. Err.     z    P>|z|  [    95% C.I.   ]      X
---------+--------------------------------------------------------------------
    ffs*|   .0971339      .03038    3.20   0.001   .037591  .156677   .395181
   good*|  -.0534889      .03637   -1.47   0.141  -.124779  .017801   .301205
fairpoor*|  -.0167317      .04362   -0.38   0.701  -.102229  .068766   .181928
main_c~c*|   .0863715      .03739    2.31   0.021   .013085  .159658   .227711
main_t~y*|   .0600158      .04087    1.47   0.142  -.020087  .140119   .180723
main_b~h*|  -.0539923       .0469   -1.15   0.250  -.145913  .037928   .163855
main_n~e*|   .1292895      .05906    2.19   0.029   .013527  .245052   .051807
comorb~1*|  -.0918458      .06924   -1.33   0.185  -.227552  .043861   .159036
comorb~2*|  -.1835928      .07168   -2.56   0.010  -.324087 -.043099   .145783
comorb~3*|  -.2584879      .07004   -3.69   0.000  -.395763 -.121213   .160241
comorb~4*|  -.2815814      .07318   -3.85   0.000  -.425003  -.13816   .137349
comorb~5*|  -.4225027      .07727   -5.47   0.000  -.573953 -.271052   .086747
como~6up*|  -.4402538       .0716   -6.15   0.000  -.580585 -.299923    .13253
  ageg1*|  -.1737429      .07134   -2.44   0.015  -.313568 -.033918   .104819
  ageg2*|  -.1835602      .04689   -3.91   0.000  -.275468 -.091653   .377108
  ageg3*|  -.1431343      .04873   -2.94   0.003  -.238647 -.047622   .290361
    pars |  -.0016944      .00128   -1.33   0.184  -.004197  .000808   79.4386
 numadl3 |  -.0059998       .0112   -0.54   0.592  -.027942  .015943   1.28072
youngmom*|  -.0072835      .04175   -0.17   0.862  -.089119  .074552   .180723
 incomep |   .0000214      .00004    0.50   0.620  -.000063  .000106   1570.84
  hsgrad*|   .0427273      .03631    1.18   0.239  -.028446  .113901   .493976
somecoll*|   .0850364      .04817    1.77   0.078  -.009374  .179447   .163855
colleg~s*|   .0051852      .10516    0.05   0.961  -.200923  .211294   .021687
    ced2 |  -.0000908      .00127   -0.07   0.943  -.002579  .002397   14.3614
------------------------------------------------------------------------------
(*) dy/dx is for discrete change of dummy variable from 0 to 1
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Prob (Y = 0) = Φ (cut1 - $\beta_i X_i$) = .753

75.3% of the children are predicted to be non-users of physical therapy services at school.

```
.mfx compute, predict (outcome (1))

Marginal effects after oprobit
      y  = Pr(schphysther==1) (predict, outcome (1))
         =  .02602718
------------------------------------------------------------------------------
variable |      dy/dx    Std. Err.     z    P>|z|  [    95% C.I.   ]      X
---------+--------------------------------------------------------------------
     ffs*|  -.0060193      .00239   -2.52   0.012  -.010708 -.001331   .395181
    good*|   .0030525       .0021    1.45   0.147   -.00107  .007175   .301205
fairpoor*|   .0009786       .0025    0.39   0.696  -.003931  .005888   .181928
main_c~c*|  -.0056721      .00297   -1.91   0.056  -.011485   .00014   .227711
main_t~y*|  -.0038822      .00297   -1.31   0.192   -.00971  .001946   .180723
main_b~h*|   .0029841      .00248    1.20   0.229  -.001874  .007843   .163855
main_n~e*|  -.0097847      .00594   -1.65   0.099   -.02142  .001851   .051807
comorb~1*|   .0047765      .00328    1.46   0.145  -.001651  .011204   .159036
comorb~2*|    .008051      .00283    2.84   0.005   .002495  .013607   .145783
comorb~3*|   .0098871      .00273    3.62   0.000   .004541  .015233   .160241
comorb~4*|   .0098919      .00261    3.79   0.000   .004776  .015008   .137349
comorb~5*|   .0081378      .00275    2.96   0.003   .002755  .013521   .086747
como~6up*|   .0093372      .00278    3.36   0.001    .00389  .014785    .13253
   ageg1*|   .0075099      .00266    2.82   0.005   .002292  .012728   .104819
   ageg2*|   .0097194      .00311    3.12   0.002   .003616  .015823   .377108
   ageg3*|   .0074172      .00276    2.69   0.007   .002004  .012831   .290361
    pars |   .0001014      .00008    1.27   0.203  -.000055  .000258   79.4386
 numadl3 |    .000359      .00067    0.53   0.595  -.000963  .001681   1.28072
youngmom*|   .0004315      .00245    0.18   0.860  -.004372  .005235   .180723
 incomep |  -1.28e-06      .00000   -0.49   0.622  -6.4e-06  3.8e-06   1570.84
  hsgrad*|  -.0025554      .00225   -1.13   0.257  -.006972  .001861   .493976
somecoll*|  -.0057076      .00382   -1.50   0.135  -.013189  .001774   .163855
colleg~s*|  -.0003135      .00642   -0.05   0.961  -.012906  .012279   .021687
    ced2 |   5.43e-06      .00008    0.07   0.943  -.000143  .000154   14.3614
------------------------------------------------------------------------------
(*) dy/dx is for discrete change of dummy variable from 0 to 1
```

Prob (Y = 1) = Φ (cut2 - $\beta_i X_i$) - Φ (cut1 - $\beta_i X_i$) = .026.

2.6% of the children are predicted to be infrequent users of physical therapy services at school.

```
.mfx compute, predict (outcome (2))

Marginal effects after oprobit
      y  = Pr(schphysther==2) (predict, outcome (2))
         =  .10531783
------------------------------------------------------------------------------
variable |      dy/dx    Std. Err.     z    P>|z|  [    95% C.I.   ]      X
---------+--------------------------------------------------------------------
     ffs*|  -.0319158      .01066   -2.99   0.003  -.052815 -.011017   .395181
    good*|   .0168171      .01127    1.49   0.136  -.005274  .038908   .301205
fairpoor*|   .0053343      .01374    0.39   0.698  -.021604  .032273   .181928
main_c~c*|  -.0293091      .01362   -2.15   0.031  -.056006 -.002612   .227711
main_t~y*|  -.0202113      .01449   -1.39   0.163  -.048612  .008189   .180723
main_b~h*|   .0166798      .01393    1.20   0.231  -.010615  .043975   .163855
main_n~e*|  -.0473243      .02469   -1.92   0.055   -.09571  .001062   .051807
comorb~1*|   .0274184      .01925    1.42   0.154  -.010301  .065138   .159036
comorb~2*|   .0497497      .01671    2.98   0.003   .017003  .082497   .145783
comorb~3*|   .0647291      .01452    4.46   0.000   .036279   .09318   .160241
comorb~4*|   .0672858      .01377    4.89   0.000   .040294  .094277   .137349
comorb~5*|   .0739307      .01003    7.37   0.000    .05427  .093591   .086747
como~6up*|   .0800078      .01068    7.49   0.000   .059075  .100941    .13253
   ageg1*|   .0467501       .0162    2.89   0.004    .01499   .07851   .104819
   ageg2*|   .0550869      .01423    3.87   0.000   .027205  .082968   .377108
   ageg3*|   .0425543      .01403    3.03   0.002   .015061  .070048   .290361
    pars |    .000547      .00042    1.31   0.189  -.000268  .001362   79.4386
 numadl3 |   .0019369      .00362    0.54   0.593  -.005157  .009031   1.28072
youngmom*|   .0023386      .01334    0.18   0.861  -.023799  .028476   .180723
 incomep |  -6.92e-06      .00001   -0.50   0.620  -.000034   .00002   1570.84
  hsgrad*|  -.0137865      .01178   -1.17   0.242  -.036884  .009311   .493976
somecoll*|  -.0292048      .01775   -1.65   0.100   -.06399   .00558   .163855
colleg~s*|  -.0016836      .03434   -0.05   0.961   -.06899  .065622   .021687
    ced2 |   .0000293      .00041    0.07   0.943  -.000774  .000833   14.3614
------------------------------------------------------------------------------
(*) dy/dx is for discrete change of dummy variable from 0 to 1
```

Prob (Y = 2) = Φ (cut3 - $\beta_i X_i$) - Φ (cut2 - $\beta_i X_i$) = .105.

10.5% of the children are predicted to be regular users of physical therapy services at school.

```
 mfx compute, predict (outcome (3))

Marginal effects after oprobit
      y  = Pr(schphysther==3) (predict, outcome (3))
         =  .11569169
------------------------------------------------------------------------------
variable |      dy/dx    Std. Err.     z    P>|z|  [    95% C.I.   ]      X
---------+--------------------------------------------------------------------
     ffs*|  -.0591989      .01851   -3.20   0.001  -.095471 -.022927   .395181
    good*|   .0336193      .02331    1.44   0.149   -.01207  .079309   .301205
fairpoor*|   .0104188       .0274    0.38   0.704  -.043281  .064119   .181928
main_c~c*|  -.0513903      .02159   -2.38   0.017  -.093708 -.009073   .227711
main_t~y*|  -.0359223      .02376   -1.51   0.131  -.082486  .010642   .180723
main_b~h*|   .0343284      .03074    1.12   0.264  -.025928  .094584   .163855
main_n~e*|  -.0721805      .02959   -2.44   0.015  -.130183 -.014178   .051807
comorb~1*|   .0596509      .04721    1.26   0.206  -.032877  .152179   .159036
comorb~2*|   .1257921      .05405    2.33   0.020   .019855  .231729   .145783
comorb~3*|   .1838716      .05664    3.25   0.001   .072864  .294879   .160241
comorb~4*|   .2044038      .06132    3.33   0.001   .084222  .324585   .137349
comorb~5*|   .3404341      .07815    4.36   0.000   .187257  .493611   .086747
como~6up*|   .3509089      .07111    4.93   0.000   .211532  .490285    .13253
   ageg1*|   .1194829      .05423    2.20   0.028   .013188  .225778   .104819
   ageg2*|   .1187539      .03219    3.69   0.000   .055671  .181837   .377108
   ageg3*|   .0931628      .03355    2.78   0.005   .027401  .158924   .290361
    pars |    .001046      .00079    1.32   0.185  -.000502  .002594   79.4386
 numadl3 |   .0037039      .00691    0.54   0.592  -.009848  .017256   1.28072
youngmom*|   .0045134      .02597    0.17   0.862  -.046391  .055418   .180723
 incomep |  -.0000132      .00003   -0.50   0.620  -.000065  .000039   1570.84
  hsgrad*|  -.0263854      .02248   -1.17   0.240  -.070436  .017665   .493976
somecoll*|   -.050124      .02721   -1.84   0.065  -.103462  .003214   .163855
colleg~s*|  -.0031881       .0644   -0.05   0.961    -.1294  .123024   .021687
    ced2 |    .000056      .00078    0.07   0.943   -.00148  .001592   14.3614
------------------------------------------------------------------------------
(*) dy/dx is for discrete change of dummy variable from 0 to 1
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Prob (Y = 3) = $1 - \Phi(\text{cut3} - \beta_i X_i) = .116$.

11.6% of the children are predicted to be frequent users of physical therapy services at school.

Interpretation—Note that the marginal effects sum to zero; this follows from the requirement that the probabilities add to 1.

FFS compared to HSCSN: coefficient on FFS is negative. This means that being in FFS relative to HSCSN moves the child in the direction of being a non-user.

Y = 0 (probability of being a non-user)

1) Children enrolled in FFS are 9.7% points more likely to receive no physical therapy at school compared to those enrolled in HSCSN.

Y = 1 (probability of being an infrequent user)
2) Children enrolled in FFS are .60% points less likely to be infrequent users of physical therapy at school than children enrolled in HSCSN.

Y = 2 (probability of being a regular user)
3) Children enrolled in FFS are almost 3.2% points less likely to be regular users than children enrolled in HSCSN.

Y = 3 (probability of being a frequent user)
4) Children enrolled in FFS are 5.92% points less likely to be frequent users than children enrolled in HSCSN.

COMORBID6UP—having 6 or more comorbid conditions versus having no other comorbid conditions. The positive coefficient means that having 6 or more comorbid conditions moves the special needs child in the direction of being a frequent user of physical therapy.

Y = 0 (probability of being a non-user) MI = -.44
1) Children with 6 plus comorbid condtions are 44% points less likely to be a nonuser of physical therapy at school compared to those with no comorbid conditions.

Y = 1 (probability of being an infrequent user) MI= .009
2) Children with 6 plus comorbid conditions are .90% points more likely to be an infrequent user of physical therapy at school compared to those with no comorbid conditions.

Y = 2 (probability of being a regular user) MI = .08
3) Children with 6 plus comorbid conditions are 8% points more likely to be regular users of physical therapy at school compared to those with no comorbid conditions.

Y = 3 (probability of being a frequent user) MI=.351
4) Children with 6 plus comorbid conditions are 35.1% points more likely to be frequent users of physical therapy at school compared to those with no comorbid conditions.